

Math 20-1 Series and Sequence Unit Assignment

1. $20^{th} - a + 19d = 107$

$d = 5$

$a + (19 \times 5) = 107$

$a + 95 = 107$

$a = 107 - 95$

$a = 12$

$\therefore t_1 = 12$

2. $t_{11} = 25$

$t_{30} = 82$

Let a be the first term and d the common difference

$a + 10d = 25$

$a + 29d = 82$

$-19d = -57$

$-19 \quad -19$

$d = 3$

$a + 10d = 25$

$a + (10 \times 3) = 25$

$a + 30 = 25$

$\therefore a = 25 - 30$

$a = -5$

$\therefore$ Common difference = 3

3. $5+x, 6, 1+2x$

$6 - (5+x) = (1+2x) - 6$

$6 - 5 - x = 1 + 2x - 6$

$1 - x = 2x - 5$

$2x + x = 1 + 5$

$3x = 6$

$3 \quad 3$

$x = 2$

4. $t_1 = 3, d = 4, S_n = 465$

$S_n = \frac{n}{2} (2a + (n-1)d)$

$465 = \frac{n}{2} (2 \times 3 + (n-1)4)$

$465 \times 2 = n (6 + 4n - 4)$

$930 = n (4n + 2)$

$4n^2 + 2n - 930 = 0$

$n = \frac{-2 \pm \sqrt{2^2 + (4 \times 4 \times 930)}}{2 \times 4}$

$n = \frac{-2 \pm 122}{8}$

$n = \frac{-2+122}{8} \text{ or } \frac{-2-122}{8}$

$n = 15 \text{ or } -18$

$\therefore$ Number of terms = 15

5. $S_8 = 176, S_9 = 216$

$176 = \frac{n}{2} (2a + (n-1)d)$

$352 = \frac{8}{2} (2a + (8-1)d)$

$352 = 4 (2a + 7d)$

$352 = 8a + 28d \rightarrow 88 = 2a + 7d$

$216 = \frac{9}{2} (2a + (9-1)d)$

$48 = 2a + 8d$

$88 = 2a + 7d$

$-40 = d$

$48 = 2a + 8d$

$48 = 2a + (8 \times -40)$

$$5. \quad 48 = 2a + (8x-40)$$

$$48 = 2a - 320$$

$$2a = 48 + 320$$

$$\frac{2a}{2} = \frac{368}{2}$$

$$a = 184$$

$$\therefore a = 184$$

$$d = -40$$

$$t_1 = 184$$

$$9^{\text{th}} = a + 8d$$

$$= 184 + (8 \times -40)$$

$$= 184 - 320$$

$$= -136$$

$$t_1 + t_9 = 184 + (-136)$$

$$= 184 - 136$$

$$= 48$$

$$6. \quad 4, 12, 36, \dots, 78732$$

$$a = 4$$

$$r = \frac{12}{4} = 3$$

$$n^{\text{th}} \text{ term} = ar^{n-1}$$

GREY

$$\frac{78732}{4} = \frac{4 \times 3^{n-1}}{4}$$

$$3^{n-1} = 19683$$

$$3^{n-1} = 3^9$$

$$n-1 = 9$$

$$n = 9+1$$

$$n = 10 \text{ terms}$$

$$7. \quad t_1 = -14 = ar^3$$

$$t_{11} = 1792 = ar^{10}$$

$$ar^3 = -14$$

$$a = \frac{-14}{r^3}$$

$$ar^{10} = 1792 \quad \text{DARK}$$

$$a = \frac{1792}{r^{10}} \quad \text{GREEN}$$

$$\frac{-14}{r^3} = \frac{1792}{r^{10}}$$

$$\frac{1792r^3}{r^3} = \frac{-14r^{10}}{r^3}$$

$$\frac{1792}{-14} = \frac{-14r^7}{-14}$$

$$r^7 = -128$$

$$r^7 = -2^7$$

$$r = -2$$

$$a = \frac{-14}{r^3}$$

$$a = \frac{-14}{(-2)^3} = \frac{-14}{-8} = \frac{7}{4}$$

$$\therefore \text{Common ratio} = -2$$

$$8. \quad S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{(1 - r)}$$

$$a = 4$$

$$r = \frac{20}{4} = 5$$

$$S_n = 15624$$

Number of terms = 6.

$$n = 6$$

$$5^n = 5^6$$

$$5^n = 15625$$

$$5^n = 15624 + 1$$

or

$$15624 = 5^n - 1$$

$$15624 = (1 - 5^n) - 1$$

$$-4$$

$$15624 = 4(1 - 5^n)$$

$$1 - 5$$

$$15624 = 4(1 - 5^n)$$

$$9. a = 3 \quad S_a = 15 \quad S_b = 63$$

$$15 = 3 \frac{(1-r^2)}{1-r}$$

$$(1-r) = \frac{3-r^2}{15}$$

$$63 = 3 \frac{(1-r^2)}{1-r}$$

$$(1-r) = \frac{3-r^2}{63} \quad \text{yellow.}$$

$$\frac{3-r^2}{15} = \frac{3-r^2}{63}$$

$$45 - 45r^2 = 189 - 189r^2$$

$$45r^2 - 189r^2 + 144 = 0$$

$$15r^2 - 63r^2 + 48 = 0$$

$$r = 4/1, -0.8$$

$$10. S_n = a \frac{(r^n - 1)}{r - 1}$$

$$-2 = 2 \frac{(0.75^n - 1)}{0.75 - 1}$$

$$-1 = \frac{0.75^n - 1}{-0.25} \quad \text{RED}$$

$$+0.25 = 0.75^n - 1$$

$$0.75^n = 0.25$$

$$n = \frac{\log 0.25}{\log 0.75} = 4.818$$

$$n \approx 5M.$$